# **Lesson Objectives**

1. Modeling with Linear Functions
2. Applications with Linear Models
3. Piecewise-Defined (Linear) Functions

# **Modeling** with Linear Functions

To model a quantity that is changing at a constant rate with , the following formula may be used:

**= (constant rate of change, changing amount, or rate)·*x* + (initial amount)**

* The constant **rate** of change (changing amount) corresponds to the **slope** of the graph of *f*.
* The **initial amount** (starting or fixed amount) corresponds to the ***y*-intercept**.
* **EXAMPLE:** Brand C soup contains 889 milligrams of sodium. Find a linear function *f* that computes the number of milligrams of sodium in *x* cans of Brand C soup. [2.2-28]

1. **Define your variables**. What information are we tracking?

Let *x* = **number of cans of soup**

Let *f*(*x*) = **milligrams of sodium in *x* total cans of soup**

1. **Identify the** **initial amount (when *x* is zero)**. Initial amount = **0**

Notice the initial (or starting) amount is not explicitly stated in this problem.

Because we are counting *x* cans of soup, there isn’t fewer than **zero** cans of soup.

Also, with zero cans of soup, there’s also **zero** sodium.

So, it’s reasonable to assume that the initial amount must be zero.

1. **Identify the** **changing amount (rate)**. Changing amount = **889** (increasing)

Since each can of soup has **889** mg of sodium, then the total sodium *f*(*x*) INCREASES by that amount, 889 mg, for each can of soup *x*.

The changing amount, then is +889 mg sodium.

1. **Write the formula for the linear function.**

**(changing amount)·*x* + initial amount**

or more simply:

# **Applications** with Linear Models

* **EXAMPLE:** A 900-gallon tank is initially full of water and is being drained at a rate of 30 gallons per minute. Complete parts (a) through (d) below. [2.4.19]

1. **Write a linear function *W* that gives the gallons of water in the tank after *t* minutes.**
2. **Define your variables:**

Let *t* = **time** in minutes

Let *W*(*t*) = **gallons in tank** after *t* minutes

1. **Identify initial amount:**

When *t* = **0**, draining hasn’t started yet, so tank is full at 900 gallons.

Initial amount = **900** (keywords: 900-gallon tank is initially **full**)

1. **Identify changing amount (rate):**

The tank is **draining** at a **rate** of **30** gallons per minute.

Changing amount = **– 30** (keyword: draining, which is **decreasing**)

1. **Write the formula:** *f*(*x*) = (changing amount)·*x* + initial amount

So, the formula is:

1. **How much water is in the tank after 4 minutes?**

Use the formula you just found to compute this.

After 4 minutes means *t* = **4**:

Use ***W*(*t*) = – 30*t* + 900**, with *t* = 4.

*W*(4) = **– 30(4) + 900** = **– 120 + 900** = **780**

So, after 4 minutes, there are **780** gallons of water in the tank.

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1. **Graph function *W* and identify and interpret the intercepts. Choose the correct graph below.**

Recall from previous page that the function is: ***W*(*t*) = – 30*t* + 900**

|  |  |  |  |
| --- | --- | --- | --- |
| A. | B. | C. | D. |
|  |  |  |  |
| **Incorrect – initial amount is too low (should be 900)** | **Incorrect slope.**  **Should be – 30** | **CORRECT** | **Incorrect –graph is increasing, which is not “draining”** |

The ***t*-intercept** is where ***W*(*t*)** (same thing as *y*) must be zero (that is ***y*** = 0).

Refer to the formula found earlier:

Set *W*(*t*) equal to zero to find the *t*-intercept.

Now, solve the equation for *t*.

So, the *t*-intercept is at **.**

The ***W*-intercept** is where ***t*** = 0. Time = zero sec. (**initial** amount of water in tank).

Refer to the formula found earlier:

The *W*-intercept (or the *y*-intercept, *b*) is: .

1. **Find the domain of *W*.** (Use set-builder notation)

Remember that domain is ***x***, but with this function, the domain involves ***t*** (time).

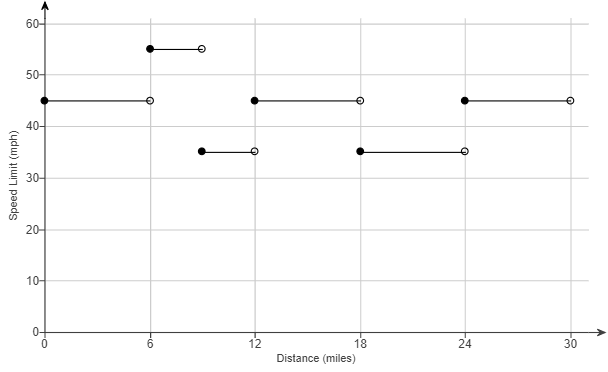
The tank **starts** draining at *t* = 0 minutes and it **stops** draining at *t* = 30 minutes.

So, the domain is: {t| **0** ≤ *t* ≤ **30** }. (the time is **between** zero and 30 minutes)

# **Piecewise-Defined** (Linear) **Functions**

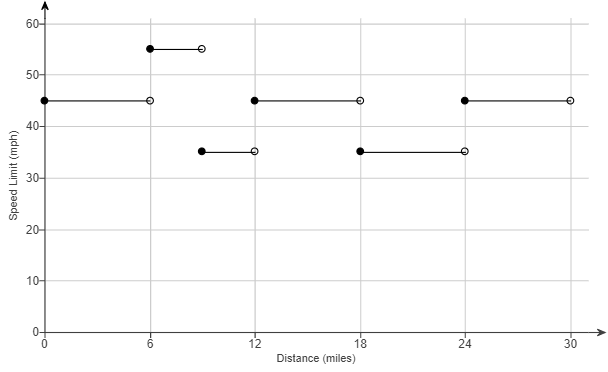
* **EXAMPLE:** The graph of gives the speed limit along a rural highway miles from its starting point. [2.4.27]

1. What are the maximum and minimum speed limits along this stretch of highway?
2. Estimate the miles of highway with a speed limit of 45 miles per hour.
3. Evaluate , , and .
4. At what -values is the graph discontinuous? Interpret each discontinuity.
5. **What are the maximum and minimum speed limits along this stretch of highway?**



Maximum speed limit = **55** mph Minimum speed limit = **35** mph

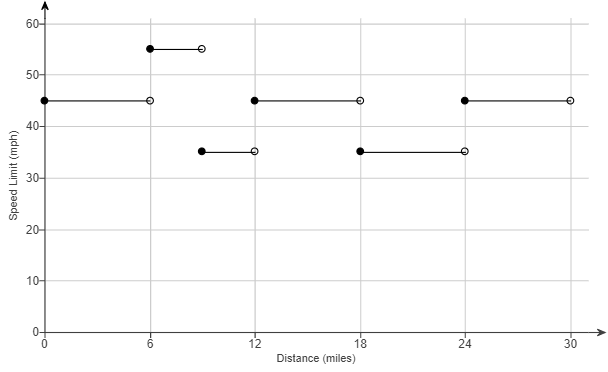
1. **Estimate the miles of highway with a speed limit of 45 miles per hour.**



Estimated miles of highway with speed of 45 mph = **3** pieces × **6** miles = **18** miles

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1. **Evaluate , , and .**



Use the **CLOSED** dot, not the open dot!

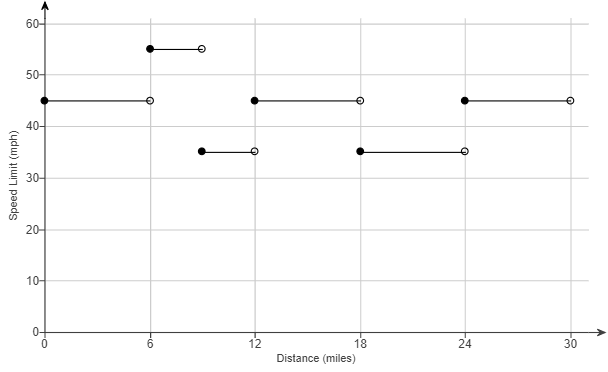
The ***y***-coordinate when *x* = 9

The ***y***-coordinate when *x* = 24

The ***y***-coordinate when *x* = 3

1. **At what -values is the graph discontinuous? Interpret each discontinuity.**

**discontinuous:** where there is a “**break**” in the graph, with another value **following** it. If you were to graph by hand, you need to **pick up** your pencil to continue the graph.



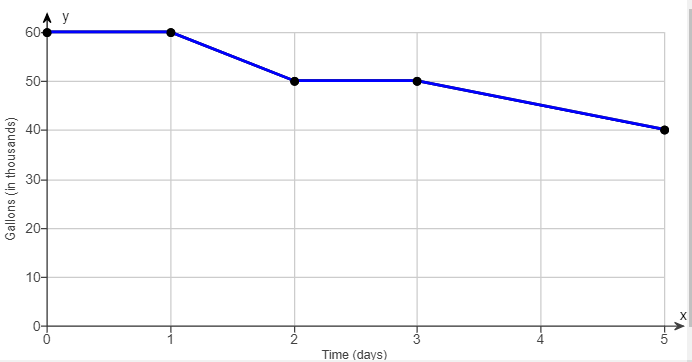
The graph is discontinuous at

(NOTE: it is NOT discontinuous at because there is nothing following it)

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* **EXAMPLE:** The graph of *y* = *f*(*x*) shows the amount of water *y* in thousands of gallons remaining in a swimming pool after *x* days. [2.4.32]

1. Estimate the initial and final amounts of water in the pool. (Type a whole number.)
2. When did the amount of water in the pool remain constant?
3. Approximate and .
4. At what rate was water being drained from the pool when ?



1. **Estimate the initial and final amounts of water in the pool. (Type a whole number.)**

Recall that the initial amount is the *y*-intercept, *b*, so find that location in the graph:

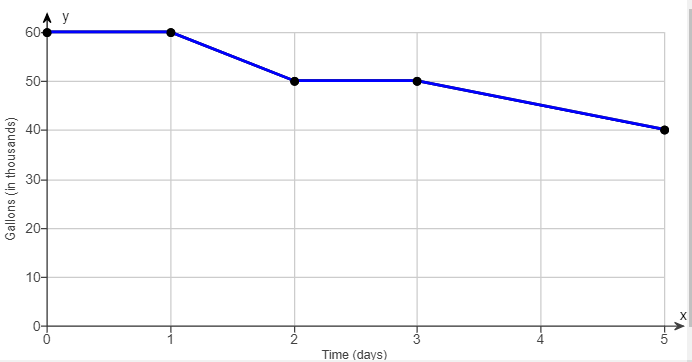
The initial amount of water in the pool was\_\_\_\_**60,000**\_\_\_\_\_\_ gallons.

The final amount is the number of gallons seen at the end (far right) of the graph:

The final amount of water in the pool was \_\_\_**40,000**\_\_\_\_\_ gallons.

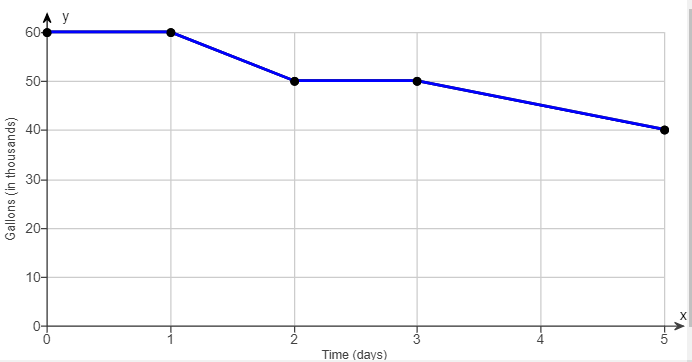
1. **When did the amount of water in the pool remain constant?**

In the graph, a **constant** amount is **horizontal** because it isn’t changing.



Choose the correct answer below

1. The amount of water in the pool was constant when and .
2. The amount of water in the pool was constant when and .
3. **The amount of water in the pool was constant when and .**
4. The amount of water in the pool was constant when and .
5. **Approximate and .**



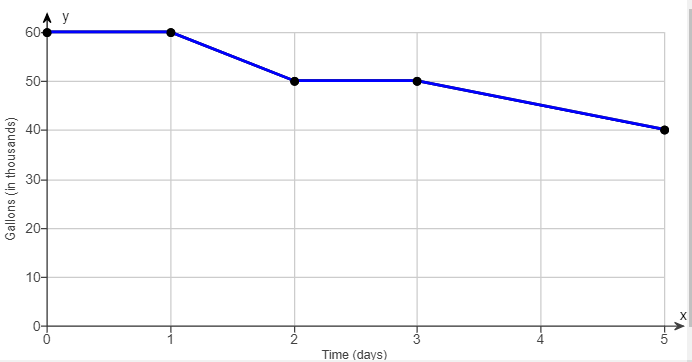
means find when . (Use the graph) \_\_**50**\_\_\_

This means that after 2 days, there are about 50,000 gallons in the pool.

means find when. (Use the graph) \_\_**45**\_\_\_

This means that after 4 days, there are about 45,000 gallons in the pool.

1. **At what rate was water being drained from the pool when ?**



When , that’s between days 3 and 5. The **rate** is its **slope**.

The water drained at a rate of **\_\_\_5000\_\_\_** gallons per day when .

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* **EXAMPLE:** For the following function find the values of

1. *G*(– 18) **(b)** *G*(3) **(c)** *G*(– 1) [\*Bittinger 2.2.97]

These are **DOMAIN** restrictions

**ALWAYS** start here **first** with **inequality** part!

* + Only **ONE** row “works” – the row you use **depends** on the **value of**  involved.
  + Work backwards – test your value for on the **RIGHT** side of each row

1. means . **Test it** in the **domain** (inequality) part first.

**TRUE** – use the **FIRST** row to plug in

**FALSE** – do **NOT** use the second row for

Using the FIRST row of the function,

1. means . **Test it** in the **domain** (inequality) part first.

**FALSE** – do **NOT** use the first row for

**TRUE** – use the **SECOND** row to plug in

Using the SECOND row of the function,

1. means . **Test it** in the **domain** (inequality) part first.

**TRUE** – use the **FIRST** row to plug in

FALSE – do NOT use the second row for

Using the FIRST row of the function,

* **EXAMPLE:** The charges for renting a moving van are $75 for the first 30 miles and $5 for each additional mile. Assume that a fraction of a mile is rounded up.

1. Determine the cost of driving the van 84 miles.
2. Find a symbolic representation for a function *f* that computes the cost of driving the van *x* miles, where .

(Hint: Express *f* as a piecewise-defined function) [\*Lial 2.6-30]

**[SOLUTION]**

|  |  |  |
| --- | --- | --- |
| (Total = **84** miles) | **Price** |  |
| First **30** miles → → → → → | $**75** |  |
| Miles remaining:  **84** – **30** = **54**  After first 30 miles, price is  $**5** each additional mile | $**5** × **54** = $**270** |  |
| Total price: | $**75** + $**270** = $**345** |  |
|  | ANSWER: **C** |  |
| Test it!  need to use **2nd** row because miles | is **TRUE** | Use **2nd** row: |

1. $6570;
2. $645;
3. $345;
4. $645;

Sources Used:

1. Pearson MyLab Math *College Algebra with Integrated Review, 12th Edition*, Lial
2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Pearson MyLab Math *Intermediate Algebra: Concepts and Applications, 10th Edition*, Bittinger